MATHEMATICS

Some Application of Trigonometry

1. Trigonometric Ratios

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

Trigonometric ratios of acute angle A in right triangle ABC are given below:

i. $\sin \angle A = \dfrac{\text{side opposite to} \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{p}{h}$

ii. $\cos \angle A = \dfrac{\text{side adjacent to} \angle A}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{b}{h}$

iii. $\tan \angle A = \dfrac{\text{side opposite to} \angle A}{\text{side adjacent to} \angle A} = \dfrac{BC}{AB} = \dfrac{p}{b}$

iv. $\operatorname{cosec} \angle A = \dfrac{\text{hypotenuse}}{\text{side opposite to} \angle A} = \dfrac{AC}{BC} = \dfrac{h}{p}$

v. $\sec \angle A = \dfrac{\text{hypotenuse}}{\text{side adjacent to} \angle A} = \dfrac{AC}{AB} = \dfrac{h}{b}$

vi. $\cot \angle A = \dfrac{\text{side adjacent to} \angle A}{\text{side opposite to} \angle A} = \dfrac{AB}{BC} = \dfrac{b}{p}$

The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

2. Relation between trigonometric ratios

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

i. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

ii. $\sec\theta = \dfrac{1}{\cos\theta}$

iii. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

iv. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

3. **Values of Trigonometric ratios** of some specific angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1
cos A	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0
tan A	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\dfrac{2}{\sqrt{3}}$	1
sec A	1	$\dfrac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined

| cot A | Not defined | $\sqrt{3}$ | 1 | $\dfrac{1}{\sqrt{3}}$ | 0 |

4. Trigonometric ratios of complementary angles

Two angles are said to complementary angles if their sum is equal to 90°. Based on this relation, the trigonometric ratios of complementary angles are given as follows:

i.   sin (90° – A) = cos A

ii.   cos (90° – A) = sin A

iii.   tan (90° – A) = cot A

iv.   cot (90° – A) = tan A

v.   sec (90° – A) = cosec A

vi.   cosec (90° – A) = sec A

Note: tan 0° = 0 = cot 90°, sec 0° = 1 = cosec 90°, sec 90°, cosec 0°, tan 90° and cot 0° are not defined.

5. **Basic trigonometric identities**:

i. $\sin^2\theta + \cos^2\theta = 1$

ii. $1 + \tan^2\theta = \sec^2\theta;\ 0 \le \theta < 90°$

iii. $1 + \cot^2\theta = \operatorname{cosec}^2\theta;\ 0 \le \theta < 90°$

6. The height or length of an object or the distance between two distant objects can be determined by the help of **trigonometric ratios**.

7. Line of sight

The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

8. Pythagoras theorem

It states that "In a right triangle, square of the hypotenuse is equal to the sum of the square of the other two sides".

When any two sides of a right triangle are given, its third side can be obtained by using Pythagoras theorem.
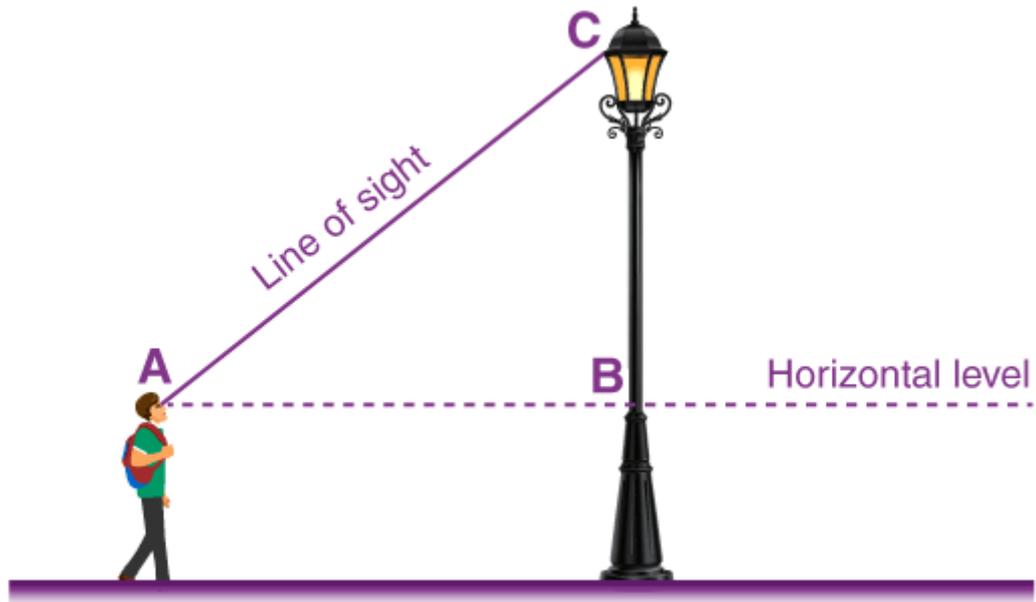
9. Reflection from the water surface

In case of reflection from the water surface, the two heights above and below the ground level are equal in length.

10. Heights and Distances

Horizontal Level and Line of Sight
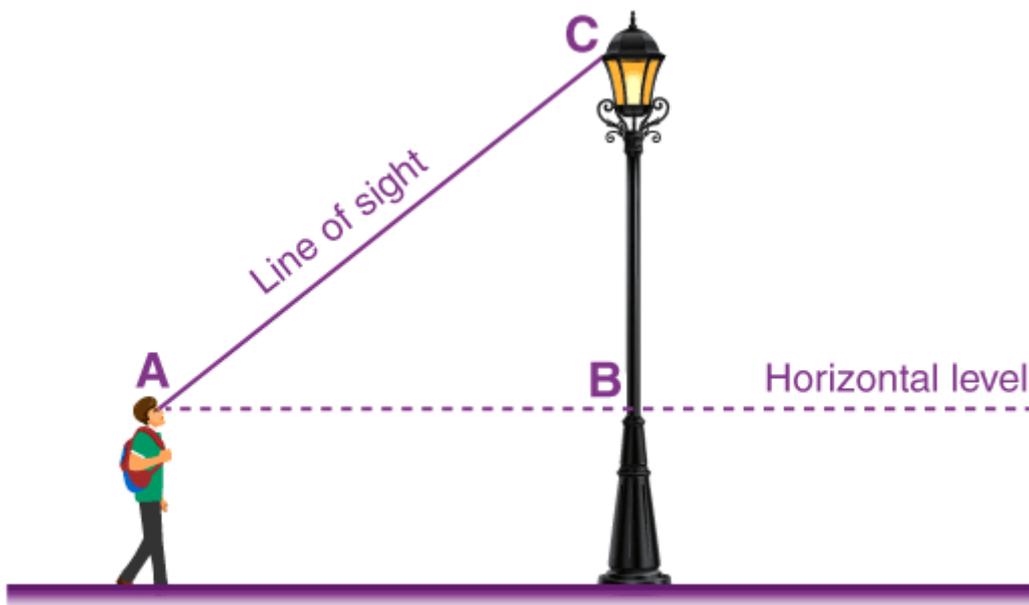
Line of sight and horizontal level

Line of sight is the line drawn from the eye of the observer to the point on the object viewed by the observer.

Horizontal level is the horizontal line through the eye of the observer.

Angle of elevation

The angle of elevation is relevant for objects above horizontal level.

It is the angle formed by the line of sight with the horizontal level.

Angle of depression

The angle of depression is relevant for objects below horizontal level.

It is the angle formed by the line of sight with the horizontal level.