MATHEMATICS

Chapter 4: Quadratic Equations

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Quadratic Equations

1. Introduction to Quadratic equation

If p(x) is a quadratic polynomial, then p(x) = 0 is called a **quadratic equation**.

The general or standard form of a quadratic equation, in the variable x, is given by $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

2. Roots of the quadratic equation

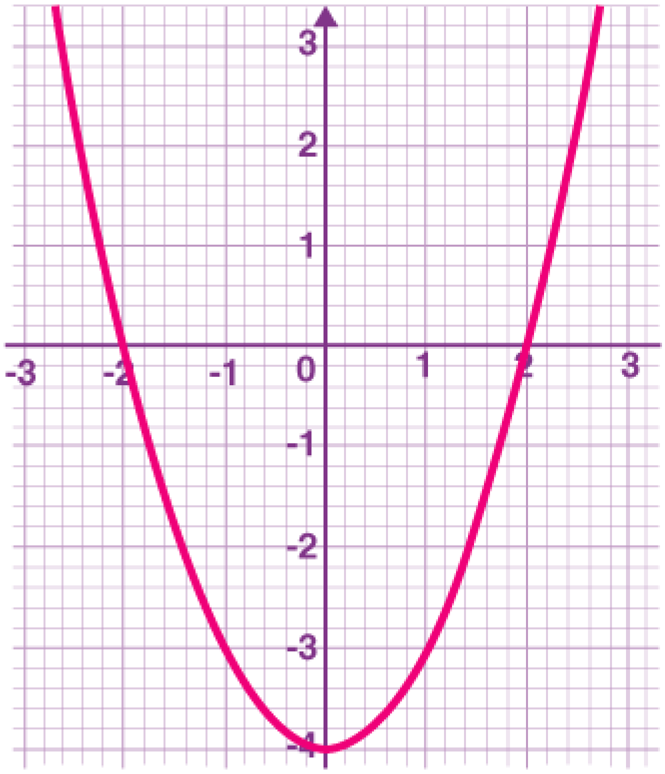
The value of x that satisfies an equation is called the **zeroes** or **roots** of the equation.

A real number α is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.

A quadratic equation has **at most two roots**.

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis.

Consider the graph of a quadratic equation $x^2 - 4 = 0$:



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation x2−4=0

Note:

- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.

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- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

3. The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

4. A quadratic equation can be solved by following algebraic methods:

i. Splitting the middle term (factorization)

ii. Completing squares

iii. Quadratic formula

5. Splitting the middle term (or factorization) method

- If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Steps involved in solving quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) by **splitting the middle term** (or factorization) method:

Step 1: Find the product ac.

Step 2: Find the factors of 'ac' that add to up to b, using the following criteria:

i. If $ac > 0$ and $b > 0$, then both the factors are positive.

ii. If $ac > 0$ and $b < 0$, then both the factors are negative.

iii. If $ac < 0$ and $b > 0$, then larger factor is positive and smaller factor is negative.

iv. If $ac < 0$ and $b < 0$, then larger factor is negative and smaller factor is positive.

Step 3: Split the middle term into two parts using the factors obtained in the above step.

Step 4: Factorize the quadratic equation obtained in the above step by grouping method. Two factors will be obtained.

Step 5: Equate each of the linear factors to zero to get the value of x.

6. Completing the square method

- Any quadratic equation can be converted to the form $(x + a)^2 - b^2 = 0$ or $(x - a)^2 + b^2 = 0$ by adding and subtracting the constant term. This method of finding the roots of quadratic equation is called the method of completing the square.
- The steps involved in solving a quadratic equation by **completing the square**, are as follows:

Step 1: Make the coefficient of x^2 unity.

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Step 2: Express the coefficient of x in the form $2 \times x \times p$.

Step 3: Add and subtract the square of p.

Step 4: Use the square identity $(a + b)^2$ or $(a – b)^2$ to obtain the quadratic equation in the required form $(x + a)^2 - b^2 = 0$ or $(x – a)^2 + b^2 = 0$.

Step 5: Take the constant term to the other side of the equation.

Step 6: Take the square root on both the sides of the obtained equation to get the roots of the given quadratic equation.

7. Quadratic formula

The roots of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ can be calculated by using the **quadratic formula:**

$\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$ where $b^2 – 4ac \geq 0$

If $b^2 – 4ac < 0$, then equation does not have real roots.

The quadratic formula is used to find the roots of a quadratic equation. This formula helps to evaluate the solution of quadratic equations replacing the factorization method. If a quadratic equation does not contain real roots, then the quadratic formula helps to find the imaginary roots of that equation. The quadratic formula is also known as Shreedhara Acharya's formula. In this article, you will learn the quadratic formula, derivation and proof of the quadratic formula, along with a video lesson and solved examples.

An algebraic expression of degree 2 is called the **quadratic equation**. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers, also called "numeric coefficients" and $a \neq 0$. Here, x is an unknown variable for which we need to find the solution. We know that the **quadratic formula** used to find the solutions (or roots) of the quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here,

a, b, c = Constants (real numbers)

$a \neq 0$

x = Unknown, i.e. variable

The above formula can also be written as:

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

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