MATHEMATICS

Chapter 3: Pair of Linear Equations in Two Variables

Pair of Linear Equations in Two Variables

1. **A pair of Linear Equations in two variables:**

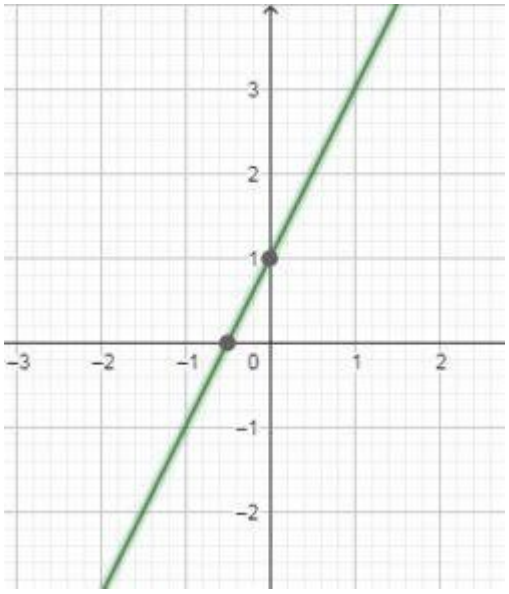
   - An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and b are not both zero, is called a **linear equation in two variables**.
   - Two linear equations in same two variables x and y are called **pair of linear equations in two variables**.

   Geometrical Representation of a Linear Equation

   Geometrically, a linear equation in two variables can be represented as a straight line.

   $2x - y + 1 = 0$

   $\Rightarrow y = 2x + 1$



   Graph of y = 2x + 1

   Plotting a Straight Line

   The graph of a linear equation in two variables is a straight line. We plot the straight line as follows:

   - Take any value for one of the variables($x_1 = 0$) and substitute it in the equation to get the corresponding value of the other variable (y_1).
   - Repeat this again (put $y_2 = 0, get\ x_2$) to get two pairs of values for the variables which represent two points on the Cartesian plane. Draw a line through the two points.

2. **Types of Polynomials based on Degree**

   Linear Polynomial

   A polynomial whose degree is one is called a linear polynomial.

   For example, 2x+1 is a linear polynomial.

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.

For example, $3x^2 + 8x + 5$ is a quadratic polynomial.

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial.

For example, $2x^3 + 5x^2 + 9x + 15$ is a cubic polynomial.

3. Graph of the polynomial x^n

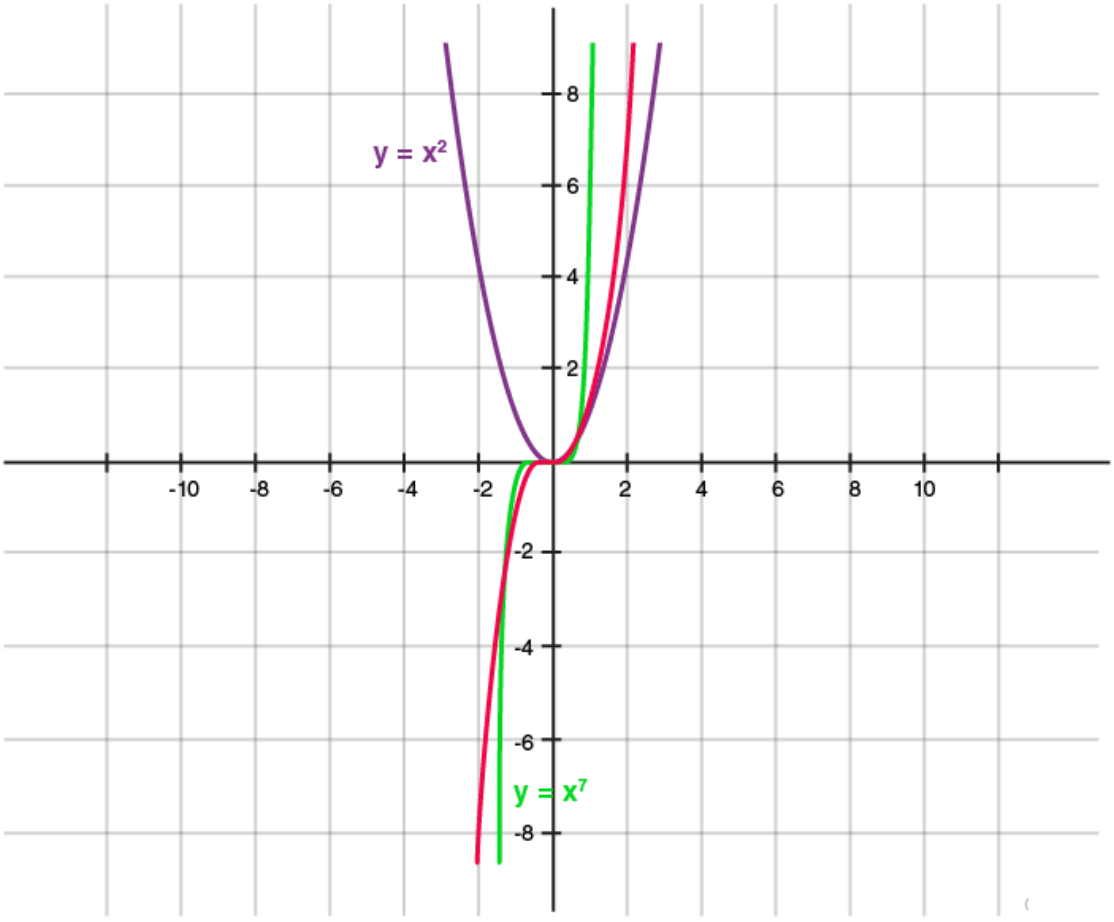
For a polynomial of the form $y = x^n$ where n is a whole number:

as n increases, the graph becomes steeper or draws closer to the Y-axis

If n is odd, the graph lies in the first and third quadrants

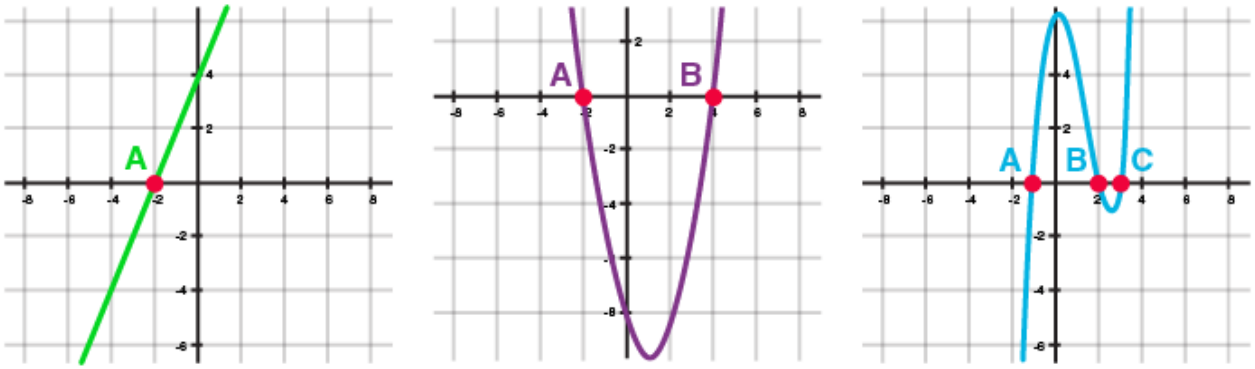
If n is even, the graph lies in the first and second quadrants

The graph of $y = -x^n$ is the reflection of the graph of $y = x^n$ on the x-axis



4. Geometrical Meaning of Zeros of a Polynomial

Geometrically, zeros of a polynomial are the points where its graph cuts the x-axis.



(i) One zero (ii) Two zeros (iii) Three zeros

Here A, B and C correspond to the zeros of the polynomial represented by the graphs.

Number of Zeros

In general, a polynomial of degree n has at most n zeros.

- A linear polynomial has one zero,
- A quadratic polynomial has at most two zeros.
- A cubic polynomial has at most 3 zeros.

5. The **general form** of a pair of linear equations in two variables is

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that

6. A system of linear equations in two variables represents two lines in a plane. For two given lines in a plane there could be three possible cases:

i. The two lines are intersecting, i. e., they **intersect at one point**.

ii. The two lines are **parallel**, i.e., they do not intersect at any real point.

iii. The two lines are **coincident** lines, i.e., one line overlaps the other line.

7. A system of simultaneous linear equations is said to be

- **Consistent**, if it has **at least one solution.**
- **In-consistent**, if it has **no solution.**

8. If the lines

i. Intersect at a point, then that point gives the **unique solution** of the system of equations. In this case system of equations is said to be **consistent**.

ii. Coincide (overlap), then the pair of equations will have **infinitely many solutions**. System of equations is said to be **consistent.**

iii. are parallel, then the pair of equations has **no solution**. In this case pair of equations is said to be **inconsistent.**