MATHEMATICS

Chapter 2: Polynomials

Polynomials

1. **What is a polynomial?**

   A **polynomial** p(x) in one variable x is an algebraic expression in x of the form $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots\ldots + a_2x^2 + a_1x + a_0$, where x is a variable

   i. a0, a1, a2 ......na are res pectively the coefficients of x0, x1, x2, x3, …… xn.

   ii. Each of a_nx^n, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$, ……… a_2x^2, a_1x, a_0, with $a_n \neq 0$, is called the term of a polynomial.

2. The highest exponent of the variable in a polynomial determines the **degree** of the polynomial.

3. Polynomials are algebraic expressions that consist of variables and coefficients. Variables are also sometimes called indeterminates. We can perform arithmetic operations such as addition, subtraction, multiplication and also positive integer exponents for polynomial expressions but not division by variable. An example of a polynomial with one variable is $x^2 + x - 12$. In this example, there are three terms: x2, x and -12.

   The word polynomial is derived from the Greek words 'poly' means 'many' and 'nominal' means 'terms', so altogether it said "many terms". A polynomial can have any number of terms but not infinite. Learn about degree, terms, types, properties, polynomial functions in this article.
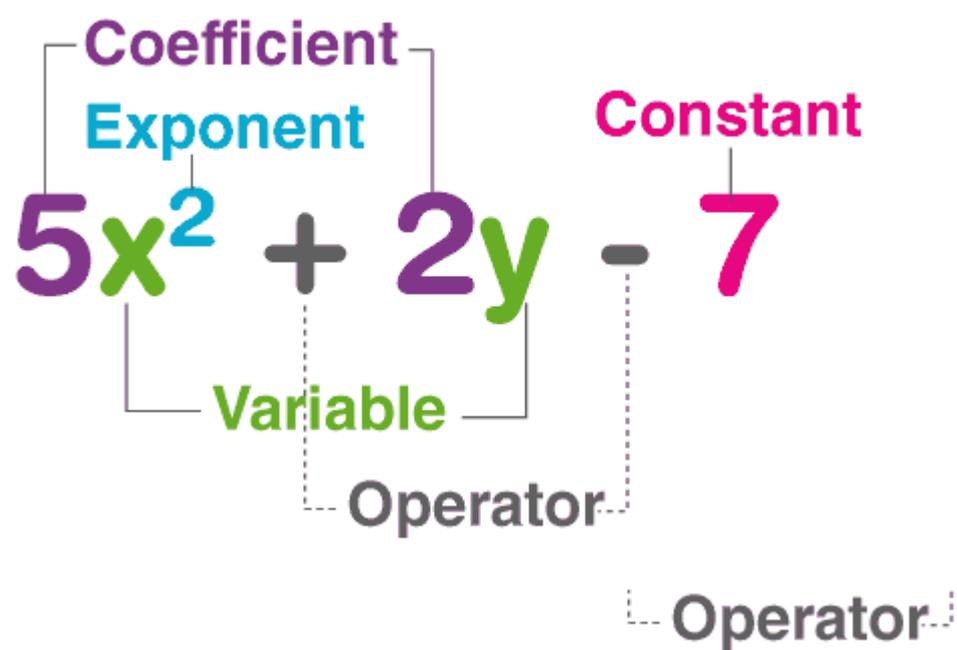
4. **Types of polynomials**

   i. A polynomial of degree zero is called a **constant polynomial**. Examples: $-9x^0, \frac{8}{14}$.

ii. A polynomial of degree one is called a **linear polynomial**. It is of the form ax + b.

   Examples: x – 2, 4y + 89, 3x – z.

iii. A polynomial of degree two is called a **quadratic polynomial**. It is of the form ax^2 + bx + c, where a, b, c are real numbers and a ≠ 0.

   Examples: x^2 - 2x + 5, x^2 – 3x etc.

iv. A polynomial of degree 3 is called a **cubic polynomial** and has the general form $ax^3 + bx^2$ + cx + d.

   For example: $x^3 + 2x^2$ - 2x + 5 etc.

Monomial: A monomial is an expression which contains only one term. For an expression to be a monomial, the single term should be a non-zero term. A few examples of monomials are:

- 5x
- 3
- $6a^4$
- -3xy

Binomial: A binomial is a polynomial expression which contains exactly two terms. A binomial can be considered as a sum or difference between two or more monomials. A few examples of binomials are:

- - 5x + 3,
- $6a^4$ + 17x
- xy^2 + xy

Trinomial

A trinomial is an expression which is composed of exactly three terms. A few examples of trinomial expressions are:

- - $8a^4$ + 2x + 7
- $4x^2$ + 9x + 7

5. **Value of the polynomial**

If p(x) is a polynomial in x, and k is a real number then the value obtained after replacing x by k in p(x) is called the value of p(x) at x = k which is denoted by p(k).

6. **Zero of a polynomial**

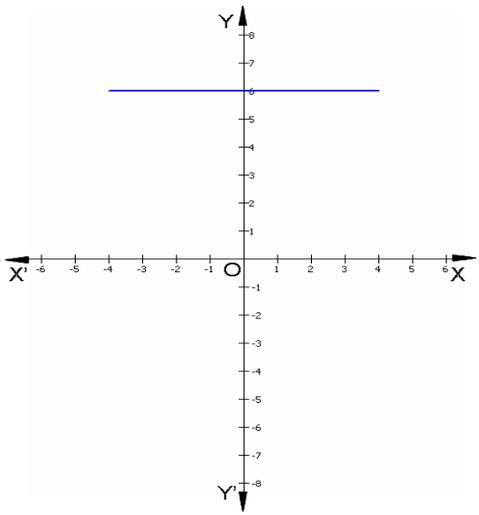- A real number k is said to be the **zero** of the polynomial p(x), if p(k) = 0.
- Zeroes of the polynomial can be obtained by solving the equation p(x) = 0.
- It is possible that a polynomial may not have a real zero at all.

- For any linear polynomial ax + b, the zero is given by the expression (-b/a) = - (constant term)/(Coefficient of x).

7. **Number of zeroes of a polynomial**

- The number of real zeros of the polynomial is the number of times its graph touches or intersects x- axis.

- The graph of a polynomial p(x) of degree n intersects or touches the x-axis at at most n points.

- A polynomial of degree n has at most **n distinct real zeroes.**

8. **A linear polynomial has at most one real zero**.

Linear Polynomial having **no zero**.	
Linear Polynomial having **one zero**.	
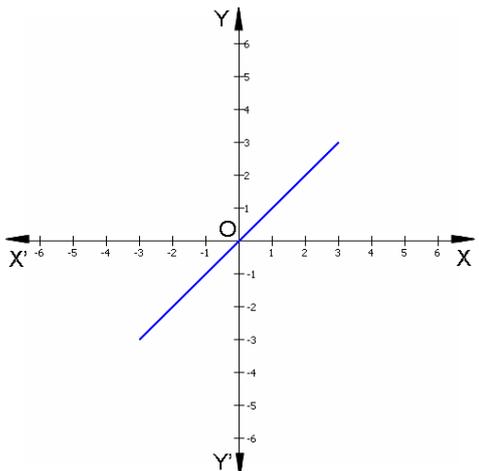
9. **A quadratic polynomial has at most two real zeroes.**