# MATHEMATICS

## Chapter 11: Constructions

# Constructions

1. **To divide a line segment internally in a given ratio m: n**, where both m and n are positive integers, we follow the steps given below:

Step 1: Draw a line segment AB of given length by using a ruler.

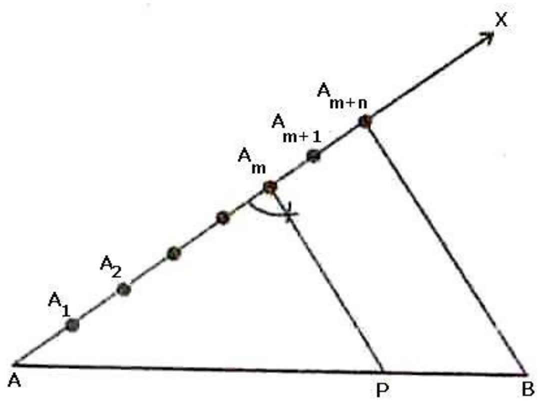
Step 2: Draw any ray AX making an acute angle with AB.

Step 3: Along AX mark off (m + n) points $A_1$, $A_2$,………$A_{m-1}$, $A_{m+1}$,………,$A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1}\,A_{m+n}$.

Step 4: Join $BA_{m+n}$

Step 5: Through the point $A_m$, draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ at $A_m$, intersecting AB at point P.

The point P so obtained is the required point which divides AB internally in the ratio m: n.




**Justification**

In $\Delta ABA_{m+n}$, we observe that $A_m$ P is parallel to $A_{m+n}$ B . Therefore, by Basic Proportionality theorem, we have:

$$\frac{AA_m}{A_mA_{m+n}} = \frac{AP}{PB}$$

$$= \frac{AP}{PB} = \frac{m}{n}\left[\because \frac{AA_m}{A_mA_{m+n}} = \frac{m}{n}, \text{by construction}\right]$$

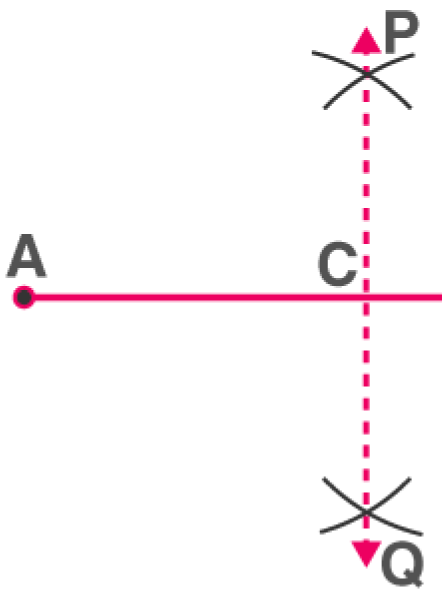
$$= AP : PB \;=\; m : n$$

Hence, P divides AB in the ratio m: n.

**Bisecting a Line Segment**

**Step 1:** With a radius of more than half the length of the line segment, draw arcs centred at either end of the line segment so that they intersect on either side of the line segment.

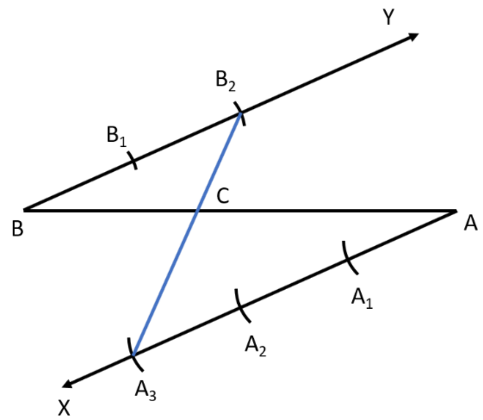
**Step 2:** Join the points of intersection. The line segment is bisected by the line segment joining the points of intersection.




PQ is the perpendicular bisector of AB

2. **Alternative method to divide a line segment internally in a given ratio m: n**

Example Find the point C such that it divides BA in ratio 2:3




**Steps of Construction:**

- Draw any ray XA making an acute angle with BA.
- Draw a ray YB parallel to XA by making ∠YBA equal to ∠XAB.
- Locate the points A1, A2, A3 (m = 3) on AX and B1, B2 (n = 2) on BY such that AA1 = A1A2 = A2A3 = BB1 =B1B2.
- Join A3B2. Let it intersect AB at a point C Then BC : CA = 2:3

**Justification**

Here $\Delta BB_2C \sim AA_3C$ ...AA test

$$\frac{BB_2}{AA_3} = \frac{BC}{AC} \ldots\ldots(\text{c.p.s.t.})$$

$$\frac{2}{3} = \frac{BC}{AC}$$

3. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as a **scale factor**. The scale factor may be less or greater than 1.
4. If the scale factor is less than 1, then the new figure will be smaller in comparison to the given figure.
5. If the scale factor is greater than 1, then the new figure will be bigger in comparison to the given figure.

## Construction of Triangle Similar to given Triangle

Consider a triangle ABC. Let us construct a triangle similar to ΔABC such that each of its sides is | | of $\left(\frac{m}{n}\right)^{th}$ the corresponding sides of □ ABC.

**Steps of constructions when m < n**:

Step 1: Construct the given triangle ABC by using the given data.

Step 2: Take any one of the three side of the given triangle as base. Let AB be the base of the given triangle.

Step 3: At one end, say A, of base AB. Construct an acute angle □BAX below the base AB.

Step 4: Along AX mark off n points $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ such that

$AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{n-1} A_n$

Step 5: Join $A_nB$

Step 6: Draw $A_mB'$ parallel to $A_nB$ which meets AB at B'.

Step 7: From B' draw B'C'| |BC meeting AC at C'.

Triangle AB'C' is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of ΔABC.