MATHEMATICS

Chapter 1: Real Numbers

Real Numbers

Euclid's Division Lemma:

Given positive integers a and b, there exists unique integers q and r satisfying $a = bq + r$, where $0 \leq r < b$

- **Lemma** is a proven statement used for proving another statement.

Euclid's Division Lemma states that given two integers a and b, there exists a unique pair of integers q and r such that $a = b \times q + r$ and $0 \leq r < b$.

This lemma is essentially equivalent to: dividend = divisor × quotient + remainder

In other words, for a given pair of dividend and divisor, the quotient and remainder obtained are going to be unique.

Euclid's Division Algorithm:

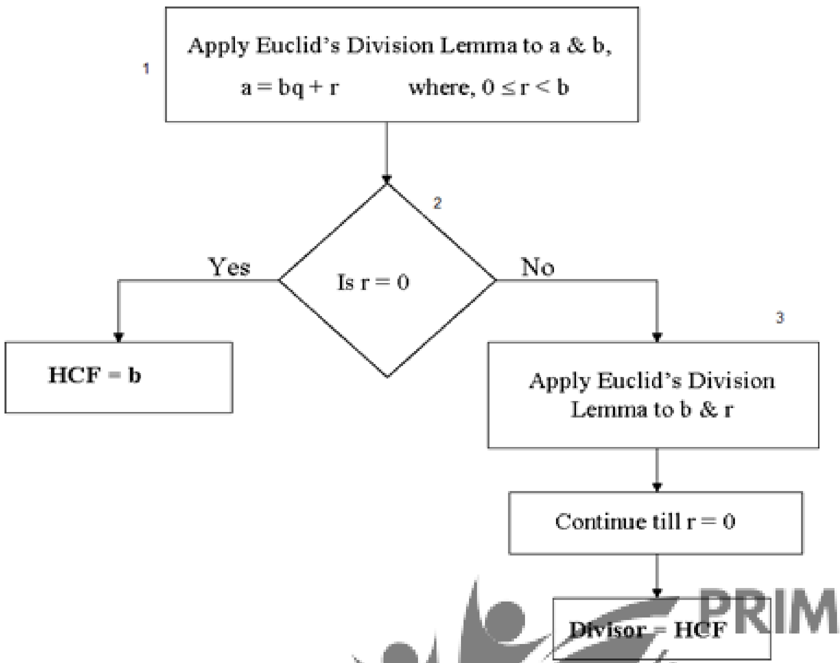
- An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problem.
- This algorithm is a technique to compute the **H.C.F** of two given positive integers.
- According to this algorithm, the **HCF** of any two positive integers 'a' and 'b', with a > b, is obtained by following the steps given below:

Step 1: Apply Euclid's division lemma, to 'a' and 'b', to find q and r, such that $a = bq + r$, $0 \leq r < b$.

Step 2: If $r = 0$, the HCF is b. If $r \neq 0$, apply Euclid's division lemma to b and r.

Step 3: Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b). Also, note that HCF (a, b) = HCF (b, r).

Euclid's Division Algorithm can be summarized as follows:



- Euclid's Division Algorithm is stated for only positive integers but it can be extended for all integers except zero, i.e., $b \neq 0$.

Consider two numbers 78 and 980 and we need to find the HCF of these numbers. To do this, we choose the largest integer first, i.e. 980 and then according to Euclid Division Lemma, $a = bq + r$ where $0 \leq r < b$;

$980 = 78 \times 12 + 44$

Now, here $a = 980$, $b = 78$, $q = 12$ and $r = 44$.

Now consider the divisor 78 and the remainder 44, apply Euclid division lemma again.

$78 = 44 \times 1 + 34$

Similarly, consider the divisor 44 and the remainder 34, apply Euclid division lemma to 44 and 34.

$44 = 34 \times 1 + 10$

Following the same procedure again,

$34 = 10 \times 3 + 4$

$10 = 4 \times 2 + 2$

$4 = 2 \times 2 + 0$

As we see that the remainder has become zero, therefore, proceeding further is not possible. Hence, the HCF is the divisor b left in the last step. We can conclude that the HCF of 980 and 78 is 2.

Let us try another example to find the HCF of two numbers 250 and 75. Here, the larger the integer is 250, therefore, by applying Euclid Division Lemma $a = bq + r$ where $0 \leq r < b$, we have

$a = 250$ and $b = 75$

$\Rightarrow 250 = 75 \times 3 + 25$

By applying the Euclid's Division Algorithm to 75 and 25, we have:

$75 = 25 \times 3 + 0$

As the remainder becomes zero, we cannot proceed further. According to the algorithm, in this case, the divisor is 25. Hence, the HCF of 250 and 75 is 25.

Real Numbers:

- The numbers which can be represented in the form $\frac{p}{q}$, of where p and q are integers and $q \neq 0$ are called **Rational numbers**.
- Any number that cannot be expressed in the $\frac{p}{q}$, form of , where p and q are integers and $q \neq 0$ are called **Irrational numbers**.
- There are more irrational numbers than rational numbers between two consecutive numbers.

- Rational and Irrational numbers together constitute **Real numbers**.

Properties of Irrational numbers:

i. The **Sum, Difference, Product** and **Division** of two irrational numbers need not always be an irrational number.

ii. **Negative** of an irrational number is an irrational number.

iii. **Sum** of a **rational** and an **irrational** number is irrational.

iv. **Product** and **Division** of a non-zero rational and irrational number is always irrational.

Fractions:

- **Terminating fractions** are the fractions which leaves remainder 0 on normal division.
- **Recurring fractions** are the fractions which never leave a remainder 0 on normal division.

Properties related to prime numbers:

- If p is a prime and divides a^2, then p divides a, where 'a' is a positive integer.
- If p is a prime, then $\sqrt{p}$ is an irrational number.
- A number ends with the digit zero if and only if it has 2 and 5 as two of its prime factors.

1. **Decimal Expansion:**

- The decimal expansion of rational number is either **terminating** or **non-terminating recurring (repeating).**
- If the decimal expansion of rational number **terminates**, then we can express the number in the form of $\frac{p}{q}$, where p and q are co-prime, and the prime factorization of **q is of the form 2^n5^m**, where n and m are non negative integers.
- If $x = \frac{p}{q}$ is a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which **terminates.**
- If the denominator of a rational number is of the form 2^n5^m, then it will terminate after n places if n > m or after m places if m > n.
- The decimal expansion of an irrational number is **non-terminating, non-recurring.**

Fundamental Theorem of Arithmetic:

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

The procedure of finding **HCF (Highest Common Factor)** and **LCM (Lowest Common Multiple)** of given two positive integers a and b: